Discretization and Stabilization of Energy-Based Controller for Period Switching Control and Flexible Scheduling

American Control Conference (ACC 2022)

Seyed Amir Tafrishi [1], Xiaotian Dai [2], Yasuhisa Hirata [1] and Alan Burns [2]

[1]Tohoku University, Japan [2]University of York, UK

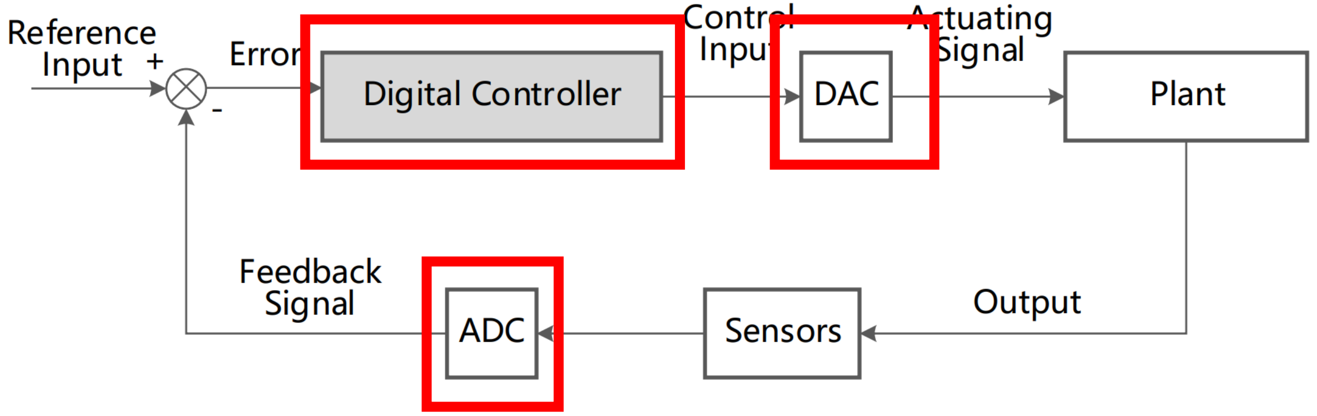
Contact: s.a.tafrishi@srd.mech.tohoku.ac.jp

TOHOKU UNIVERSITY

UNIVERSITY of York

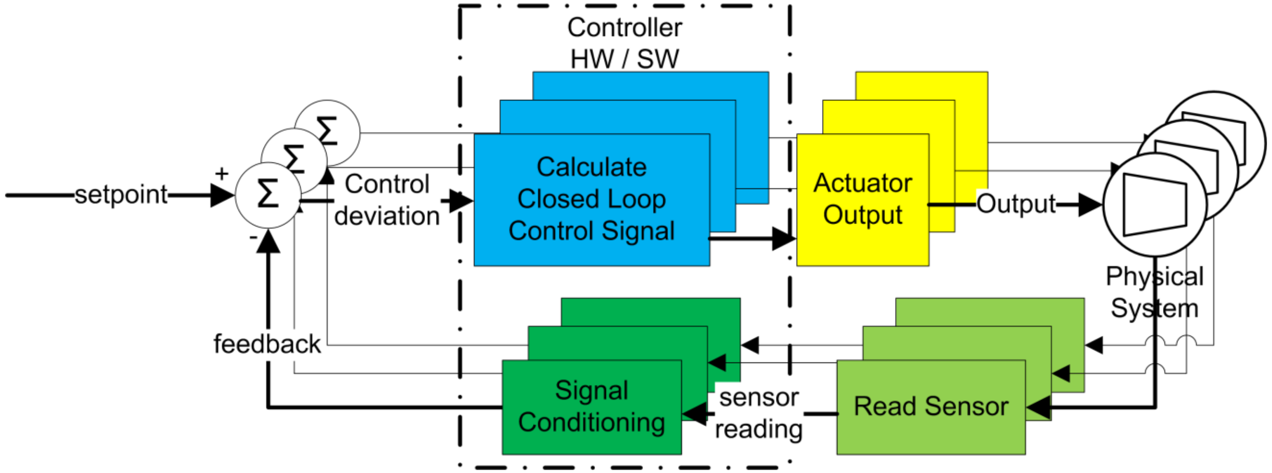
Introduction

- Digital controllers



- In classic methods the **sampling time** kept constant

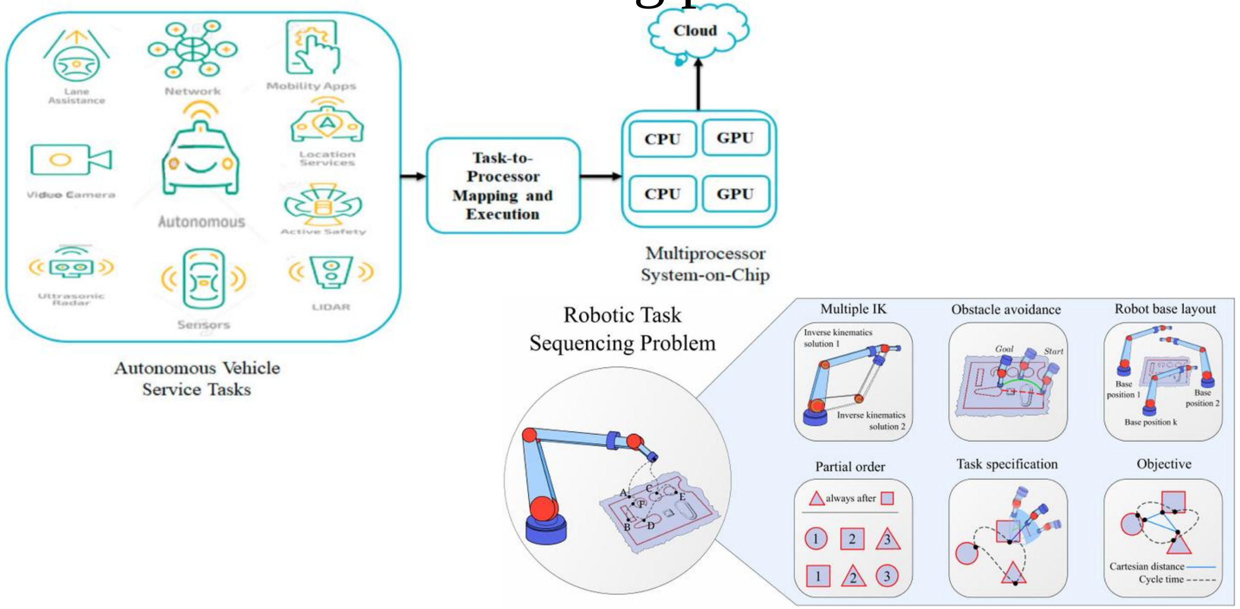
- This brings serious **limitations** in:
  - Robustness and abilities of the controller
  - In multi-systems with limited masters when different operating frequencies exist
  - Scheduling and random non-uniform sampling



Introduction

- Nonuniform sampling is important in actuators and task-scheduling problems

Actuators and robot mechanisms



Digital task scheduling in automation

- For instance, **PWM control** is conventionally used in driving the motors **but**
  - Considerable voltage spikes in high-frequency controls
  - Large energy losses in high-frequency
  - Magnetic noise and disturbances

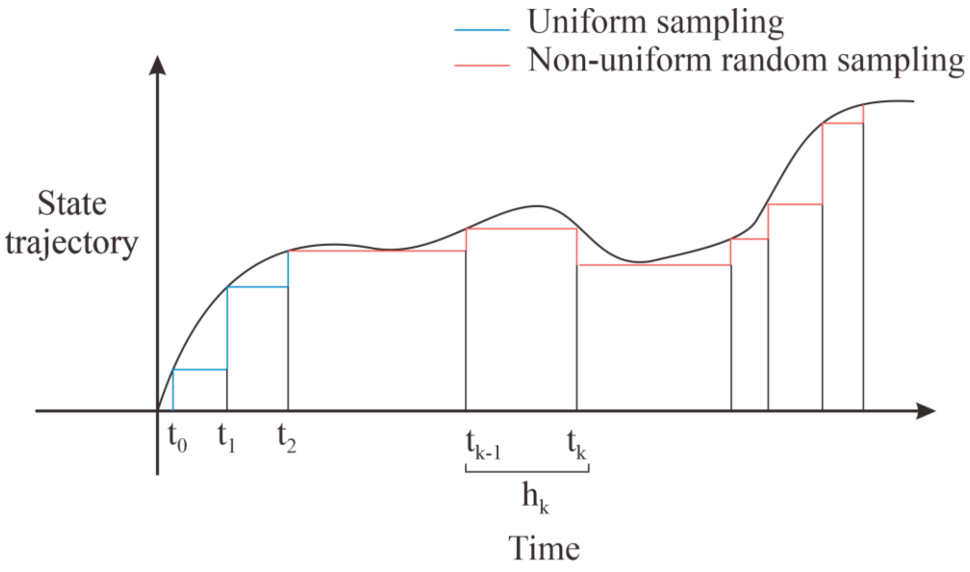
Non-uniform random sampling controllers will make more efficient discrete-time controllers

Problem statement

- The system has a **random non-uniform sampling time** ($h_k = t_k - t_{k-1}$) values, changing time length

- The **controller** should **converge** the **states** $\mathbf{x}_k$ to desired states $\mathbf{x}_d$

- The controller should be **robust** enough with the ability to **stabilize** the system while sampling time changes continuously
(the **sampling time** itself becomes a **disturbance**)



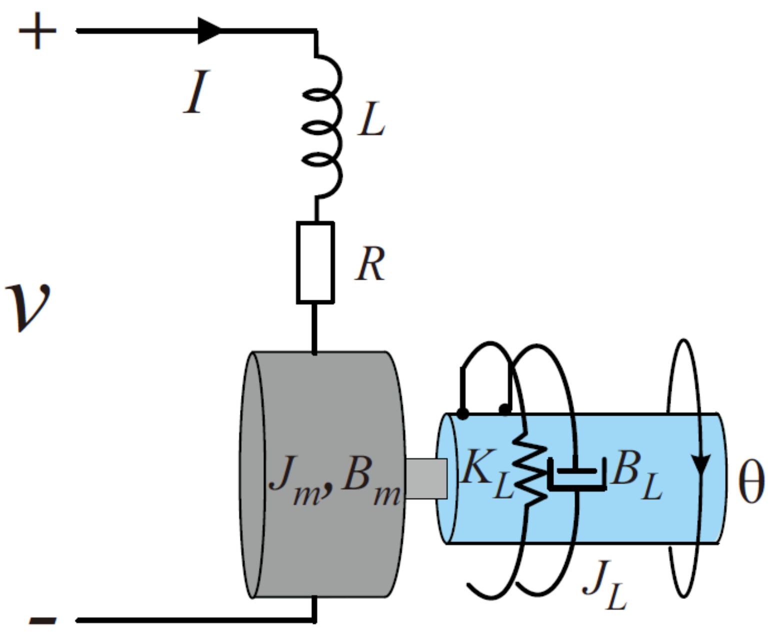
Motor model with a flexible load

State-space model in continues-time

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u,$$
$$\mathbf{y} = \mathbf{C}\mathbf{x},$$

where

$$\mathbf{A} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} & 0 \\ \frac{K_m}{J} & -\frac{B}{J} & -\frac{K_L}{J} \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -\frac{1}{L} \\ 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$



where states are $\mathbf{x} = [I \quad \dot{\theta} \quad \theta]^T$ and

$J = J_m + J_L$: Motor and load inertia

$B = B_m + B_L$: Motor and load viscous friction

K_L : Load stiffness

Discrete state-space model

The discretized model is defined with sampling time h_k as

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{G}_k \mathbf{u}_k,$$

where

$$\mathbf{F}_k := e^{\mathbf{A}h_k}, \quad \mathbf{G}_k := \int_0^{h_k} e^{\mathbf{A}h_k}\, \mathbf{B}\, d\tau,$$

Then, the model is transformed to Maclaurin series by

$$\mathbf{\Phi}(\mathbf{A}) \triangleq \sum_{i=0}^{\infty} \frac{\mathbf{A}^i}{(i+1)!} = \mathbf{I} + \frac{\mathbf{A}}{2!} + \frac{\mathbf{A}^2}{3!} + \ldots$$

which has following properties

$$(i) \quad \mathbf{A}\mathbf{\Phi}(\mathbf{A}) = \mathbf{\Phi}(\mathbf{A})\mathbf{A},$$

$$(ii) \quad e^{\mathbf{A}} = \mathbf{I} + \mathbf{A}\mathbf{\Phi}(\mathbf{A}),$$

$$(iii) \int_0^h e^{\mathbf{A}h} d\tau = h\mathbf{\Phi}(\mathbf{A}h),$$

(iv) $\mathbf{\Phi}(\mathbf{T}^{-1}\mathbf{A}\mathbf{T}) = \mathbf{T}^{-1}\mathbf{\Phi}(\mathbf{A})\mathbf{T}$ for an arbitrary invertible $\mathbf{T}$,

An energy-based controller with dynamic gain

Proposition: The motor model with random nonuniform sampling h_k will always converge with following controller input

$$u_k = -\left[k_E\left(\frac{1}{2}\mathbf{x}_k^T\mathbf{D}\mathbf{x}_k\right)\mathbf{x}_k^T\mathbf{D}\mathbf{\Phi}(\mathbf{A}h_k)\mathbf{B}\right]^{-1}\left[\left(k_E\left(\frac{1}{2}\mathbf{x}_k^T\mathbf{D}\mathbf{x}_k\right)\mathbf{x}_k^T\mathbf{D}\right)\mathbf{\Phi}(\mathbf{A}\mathbf{h_k})\mathbf{A}\mathbf{x}_k + \frac{k_D}{h_k}\left(\dot{\theta}_k - \dot{\theta}_d\right)\left(\mathbf{F}_m\mathbf{x}_k - \dot{\theta}_k\right) + k_P\left(\theta_k - \theta_d\right)\dot{\theta}_k\right]$$

k_E: Dynamic energy gain
k_P: Proportional gain
k_D: Derivative gain

Under the boundedness of the sampling time condition by

$$\begin{cases} k_D\left(\dot{\theta}_k - \dot{\theta}_d\right)\left(\mathbf{F}_m\mathbf{x}_k - \dot{\theta}_k\right) \le 0 & h_k \to 0 \\ -\mathbf{A}\mathbf{x}_k \le \mathbf{B}u_k & h_k \to \infty \end{cases}$$

And singular-free config. and physical model condition

$$k_E\left(\frac{1}{2}\mathbf{x}_k^T\mathbf{D}\mathbf{x}_k\right)\mathbf{x}_k^T\mathbf{D}\mathbf{\Phi}(\mathbf{A}h_k)\mathbf{B}u_k + k_P\left(\theta_k - \theta_d\right)\dot{\theta}_k \le -k_E E_k\,\mathbf{x}_k^T\mathbf{D}\mathbf{\Phi}(\mathbf{A}h_k)\mathbf{A}\mathbf{x}_k - \frac{k_D}{h_k}\left(\dot{\theta}_k - \dot{\theta}_d\right)\left(\mathbf{F}_m\mathbf{x}_k - \dot{\theta}_k\right)$$

*Please refer to paper for more details about the proof

Controller Lyapunov function

- The derivative of Lyapunov function ($V = \frac{1}{2}k_E E^2 + \frac{1}{2}k_D(\dot{\theta} - \dot{\theta}_d)^2 + \frac{1}{2}k_P(\theta - \theta_d)^2$) is derived and discretized using Maclaurin series and our discrete model

$$V' \approx \frac{1}{h_k}(V_{k+1} - V_k) = \frac{1}{h_k}k_E E_k\left(E_{k+1} - E_k\right) + \frac{k_D}{h_k}\left(\frac{1}{h_k}(\theta_{k+1} - \theta_k) - \dot{\theta}_d\right)\left(\dot{\theta}_{k+1} - \dot{\theta}_k\right) + \frac{k_P}{h_k}\left(\theta_k - \theta_d\right)\left(\theta_{k+1} - \theta_k\right)$$

$$\Rightarrow \quad \boxed{V' = k_E E_k\left[\mathbf{x}_k^T \mathbf{D}\mathbf{\Phi}(\mathbf{A}h_k)\left(\mathbf{A}\mathbf{x}_k + \mathbf{B}u_k\right)\right] + \frac{k_D}{h_k}\left(\dot{\theta}_k - \dot{\theta}_d\right)\left(\mathbf{F}_m\mathbf{x}_k - \dot{\theta}_k\right) + k_P\left(\theta_k - \theta_d\right)\dot{\theta}_k}$$

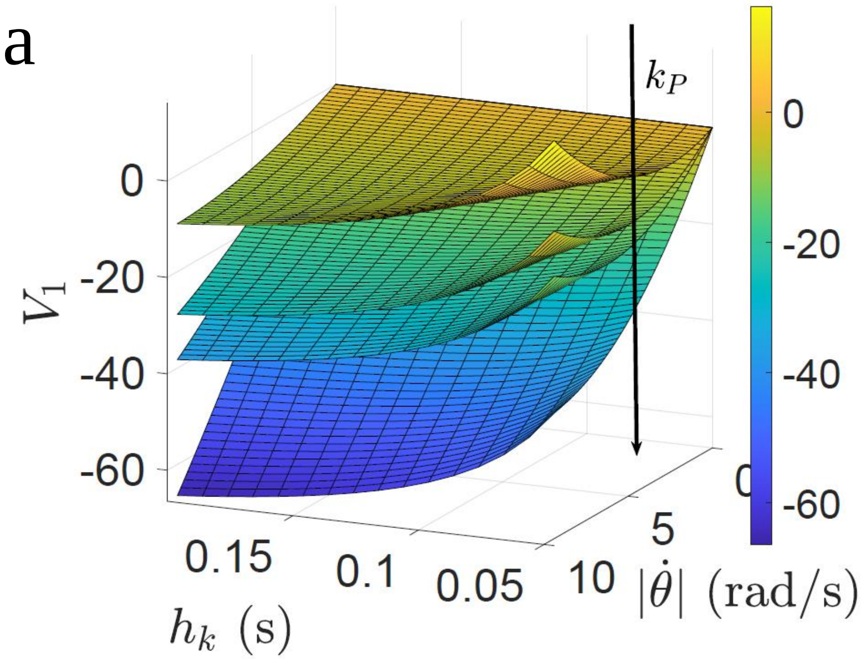
- With a positive definite Lyapunov function a stable converges can be achieved under Lasalle’s theorem

$$V'(\mathbf{x}_k) \leq 0, \quad \forall \mathbf{x}_k \in \mathbb{R}^3$$

while satisfying two inequalities:

$$V_1 : k_P\left(\theta_k - \theta_d\right)\dot{\theta}_k - \frac{k_D}{h_k}\left(\dot{\theta}_k - \dot{\theta}_d\right)\left(\mathbf{F}_m^*\mathbf{x}_k - \dot{\theta}_k\right) \leq 0$$

$$V_2 : \mathbf{B}u_k + \mathbf{A}\mathbf{x}_k \leq 0$$



Dynamic gain tuning of energy term

- **Energy term** in Lyapunov function is main factor that is **sensitive** to the **sampling time** (h_k)

- We propose a **tuning dynamic gain** based on a **standard energy**

$$k_E(h_k) = E'_s / E'(h_k) + K_c$$

where
K_c : Constant small value
E'_s: Standard discretized energy

- An inequality condition to grant stability with dynamic energy gain:

$$k_E\left(E'_s/E'(h_k)+K_c\right)\left[\mathbf{x}_k^T\mathbf{D}\mathbf{\Phi}(\mathbf{A}^*h_k)\mathbf{B}u_k\right] \leq k_E\left(E'_s/E'(h_k)+K_c\right)\ \mathbf{x}_k^T\mathbf{D}\mathbf{\Phi}(\mathbf{A}^*h_k)\mathbf{A}^*\mathbf{x}_k + \frac{k_D}{h_k}\left(\dot{\theta}_k-\dot{\theta}_d\right)\left(\mathbf{F}_m^*\mathbf{x}_k-\dot{\theta}_k\right)$$

*Please refer to paper for more details about the proof

Simulation results

- The results for three different random sampling time cases

- Initial and desired states:

  $\{\theta_0, \dot{\theta}_0, I_0\} = \{0.1, 5, 0.4\}$

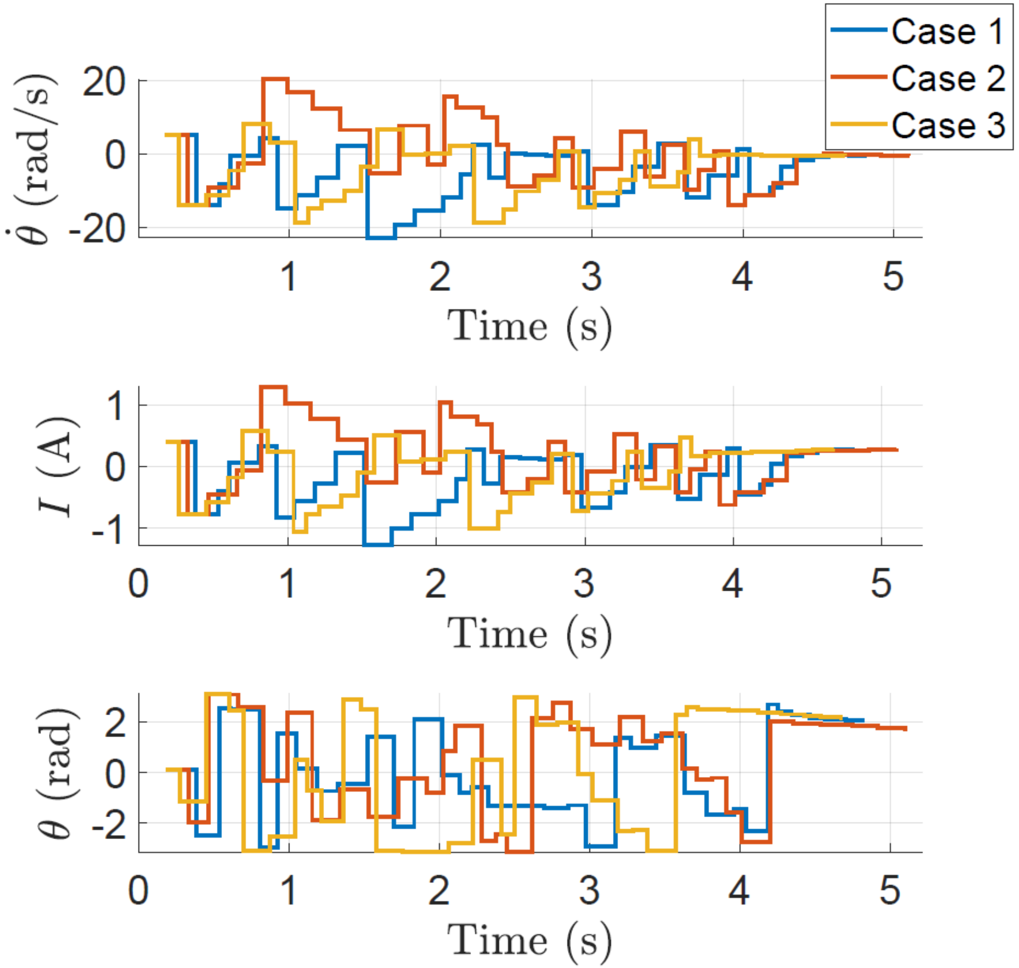
  $\{\theta_d, \dot{\theta}_d, I_d\} = \{2, 0, 0\}$

- Based on stability conditions sampling time is bounded with

  $h_k \in [h_{min}, h_{max}] = [0.05, 0.2]$ s

Variable	Value	Variable	Value
J	0.004 kg-m^2	B	0.04 Nm-s/rad
R	1.3 Ω	L	1 mH
K_b	0.5	K_m	0.5
K	0.7 N/m	k_P	565
$k_{E,s}$	725	k_D	0.07
h_s	0.11	u_{sat}	45 V
K_c	610	K_L	0.4

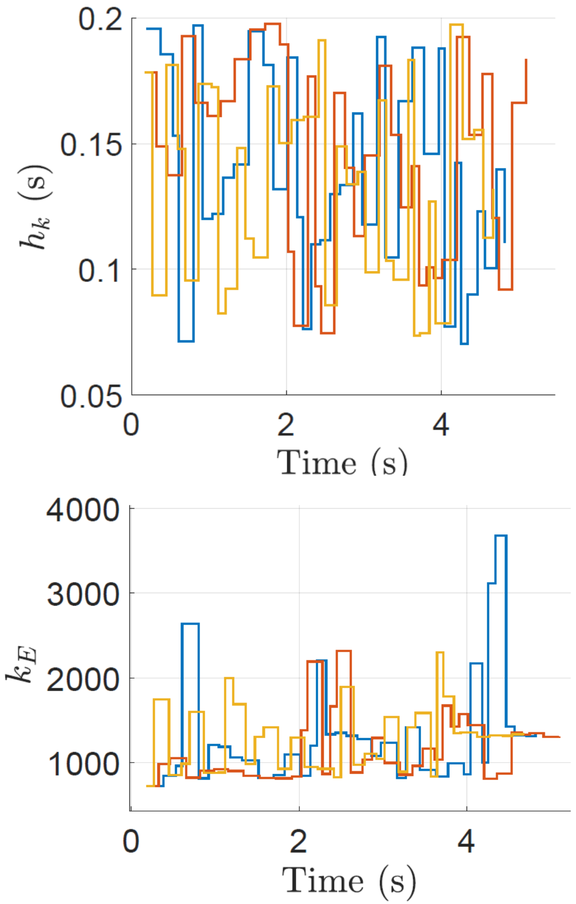
The system parameters



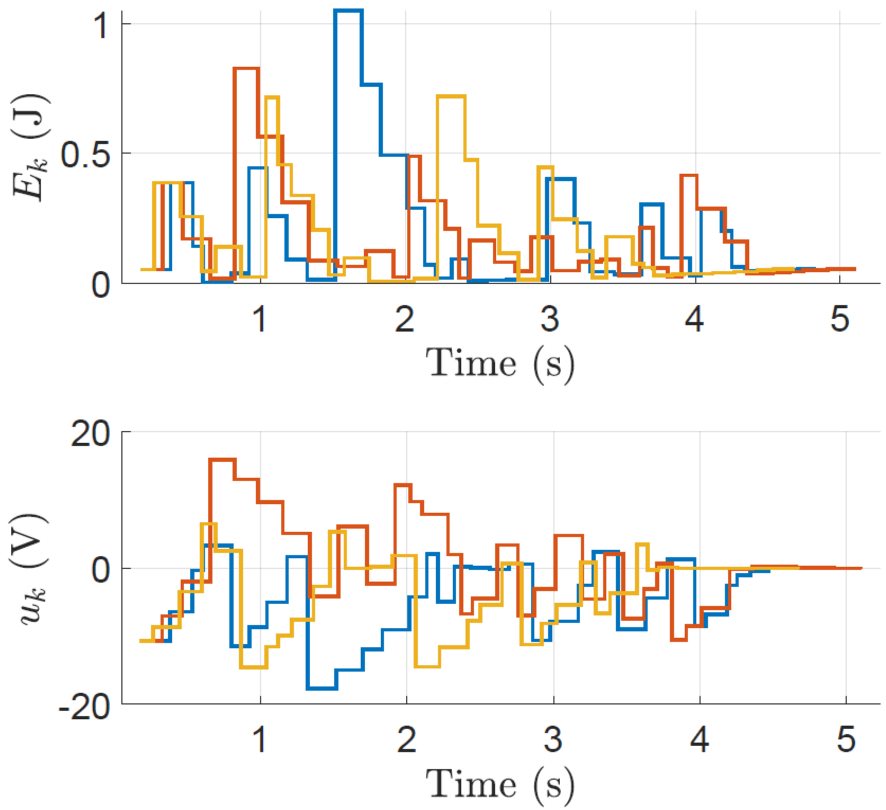
Simulation results of the system states.

Simulation results

- More details regarding the controller behavior



Sampling time and energy gain

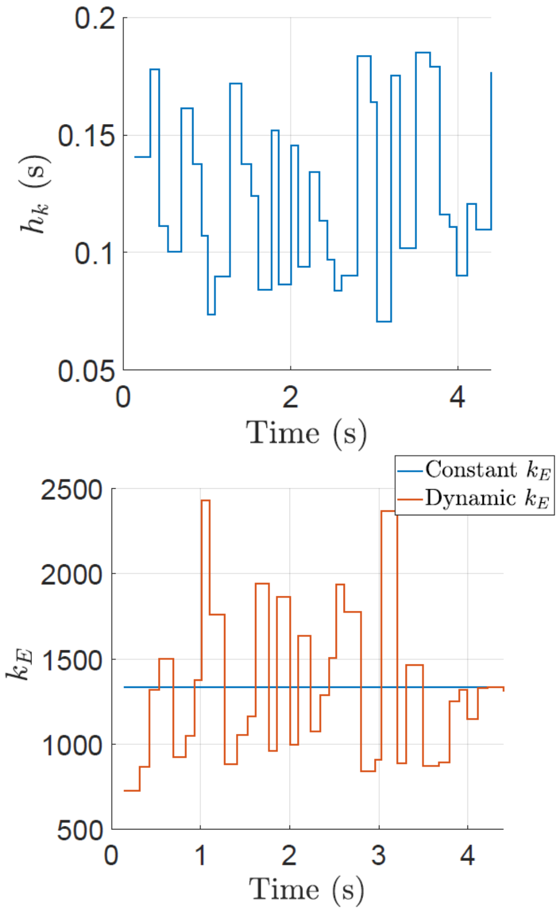


Energy of system and control input

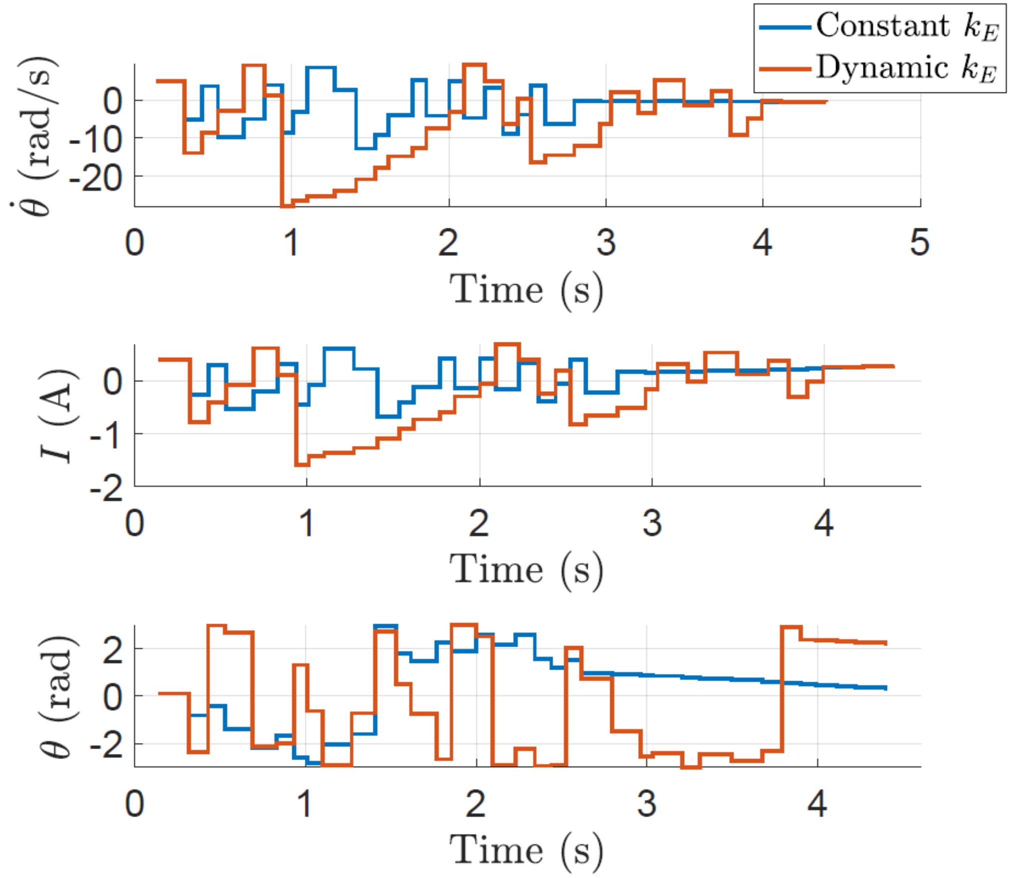
Simulation results

- The dynamic k_E effect's in system convergence

- The constant k_E controller can not converge angular state to desired values



Sampling time and energy gain



Simulation results of the system states.

Conclusion and Future Works

- Conclusion
  - An energy-based controller to converge the system with non-uniform random sampling with designed dynamic gain is proposed
  - Performance of the controller checked for the DC motor system with flexible load

- Open challenges
  - Although the **elastic load** makes the **converges longer**, better strategies should be investigated to **decrease** the **converges time**,
  - **Dependency** of the energy-based controller to **physical parameters,** based on its nature, **limits** the **application** ranges (non-physical models e.g., real-time systems)
- Future works
  - Developing a **model predictive controller** with **changing horizon**